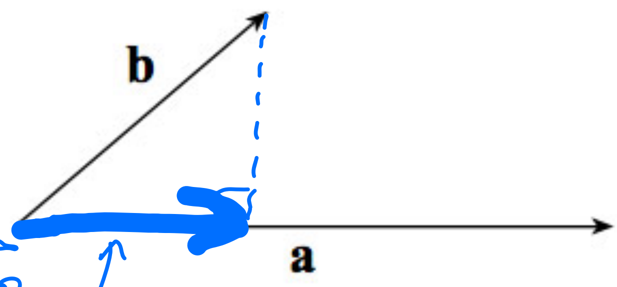
[9.3] - Projections & dot product

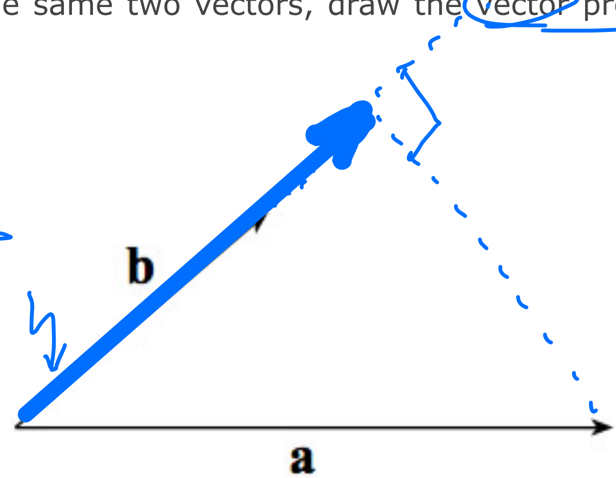
Read section 9.3 in the textbook and answer these questions.

1. For the two vectors shown, draw the vector projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$.



$\text{proj}_{\vec{a}} \vec{b}$

2. For the same two vectors, draw the vector projection of $\vec{\mathbf{a}}$ onto $\vec{\mathbf{b}}$.



$\text{proj}_{\vec{b}} \vec{a}$

3. What is the angle between the vectors $\underset{\vec{a}}{\langle 1, 0, -1\rangle}$ and $\underset{\vec{b}}{\langle 1, 1, 0\rangle}$?

$\vec{a}\cdot\vec{b} = 1^2 + 0 + 0 = 1 \qquad \vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow \cos(\theta) = \frac{\vec{a}\cdot\vec{b}}{ab} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \qquad \arccos\frac{1}{2} = 60°$

$|\vec{a}| = a = \sqrt{2} = b$

4. Suppose that $\vec{\mathbf{a}} \neq 0$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$. This means that $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ have the same projection onto $\vec{\mathbf{a}}$. Does this automatically mean that $\vec{\mathbf{b}} = \vec{\mathbf{c}}$?

   Consider $\vec{\mathbf{a}} = \langle 1, 2\rangle$ and $\vec{\mathbf{b}} = \langle 2, 1\rangle$. Show that the answer to the question above is "no", by finding a vector $\vec{\mathbf{c}}$ which is not equal to $\vec{\mathbf{b}}$, but never the less satisfies $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$.

$\vec{a}\cdot\vec{b} = \langle 1,2\rangle\cdot\langle 2,1\rangle = 1\cdot 2 + 2\cdot 1 = 4 = \vec{a}\cdot\vec{c} = \langle 1,2\rangle\cdot\langle c_x, c_y\rangle$

$4 = c_x + 2c_y$

Many possibilities! $\vec{c} = \langle \frac{4}{3}, \frac{4}{3}\rangle$ or $\langle 0, 2\rangle$ or $\langle 4, 0\rangle$ or...