Math 213 - 10.2 - Tabular Data

Assume units of wave height are *feet*.

Wave Heights on the Open Sea

The wave heights h in the open sea depend on the speed v of the wind (knots) and the length of time t that the wind has been blowing at that speed (hours). Values for the function $h = f(v,t)$ are in the following table.

v\t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

Questions:

1. What is the value of $f(40,15)$? What is its meaning?

2. What is the meanings of the function $h = f(30,t)$? $h = f(v,30)$?

3. Estimate the values of $\frac{\partial f}{\partial v}(40,20)$ and $\frac{\partial f}{\partial t}(40,20)$ and interpret their meanings.

4. Find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours. (Round the numerical coefficients to two decimal places).

5. Using the linear approximation, estimate the wave heights when the wind has been blowing for 24 hours at 43 knots. (Round the answer to two decimal places).

6. What do you think is the $\lim_{t\to\infty} \frac{\partial f}{\partial t}$?

Math 213 - - Tabular Data

10.2

Partial Derivatives and Data

The function $f(x,y)$ is given by the following data.

	x=0	x=10	x=20	x=30
y=0	89	80	74	71
y=2	93	85	80	76
y=4	98	91	85	81
y=6	104	98	92	88
y=8	112	105	99	94

What is $f(10,6)$?

If $f(x,y) = 98$ and $y = 4$ then what is x?

Estimate $\dfrac{\partial f}{\partial x}$ at $(20,4)$.

Estimate $\dfrac{\partial f}{\partial y}$ at $(20,4)$.

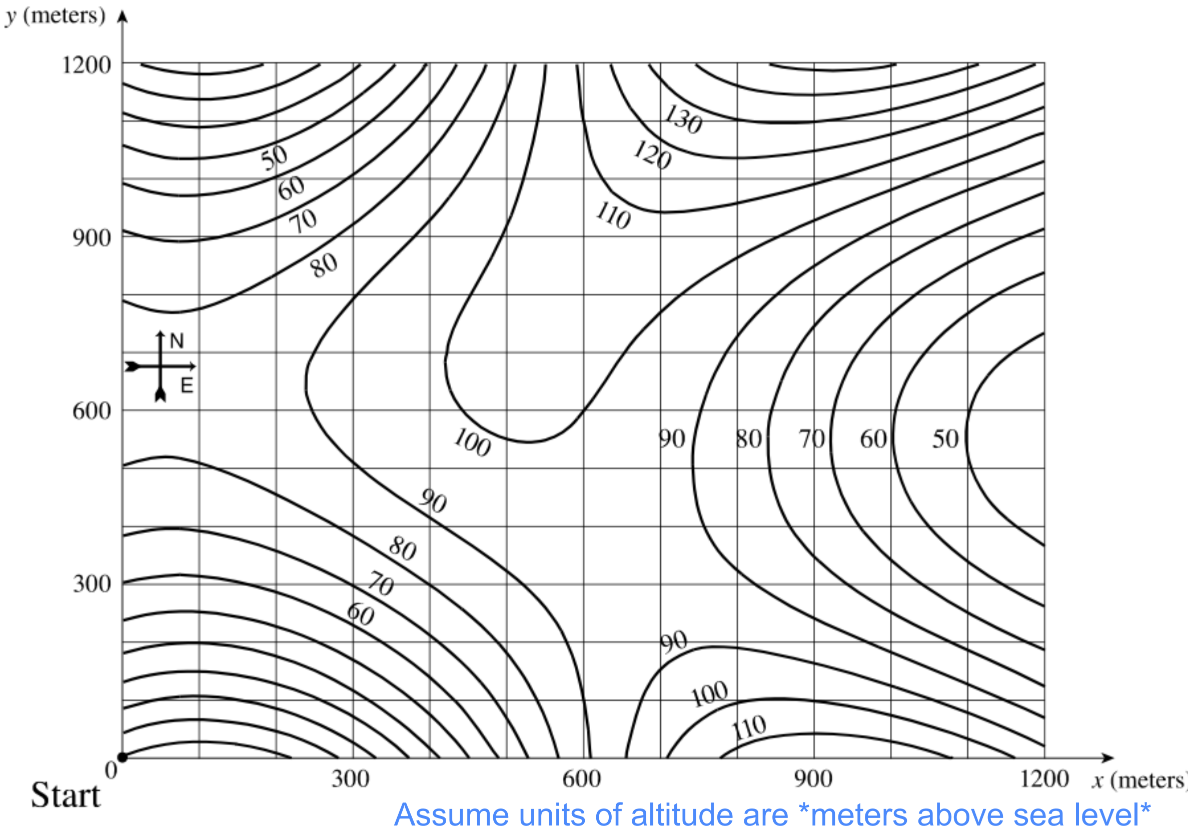
Use these partial derivatives to estimate $f(22,4)$.

Use these partial derivatives to estimate $f(20,5)$.

Estimate $f(22,5)$.

Math 213 - 10.2 - Graphical Data

The following is a map with curves of the same elevation of a region in Orangerock National Park.



Assume units of altitude are *meters above sea level*

We define the altitude function $A(x,y)$ as the altitude at a point x meters east and y meters north of the origin ("Start").

1. Estimate $A(300,300)$ and $A(500,500)$.

2. Estimate $A_x(300,300)$ and $A_y(300,300)$.

3. What do A_x and A_y represent in physical terms?

Math 213 - 10.2 - Graphical Data

4. In which direction does the altitude increase most rapidly at the point (300, 300)?

5. Use your estimates of A_x (300,300) and A_y (300,300) to approximate the altitude at (320, 310).

Math 213 - 10.2 - More Graphical Data

1. Refer to the following contour graph.

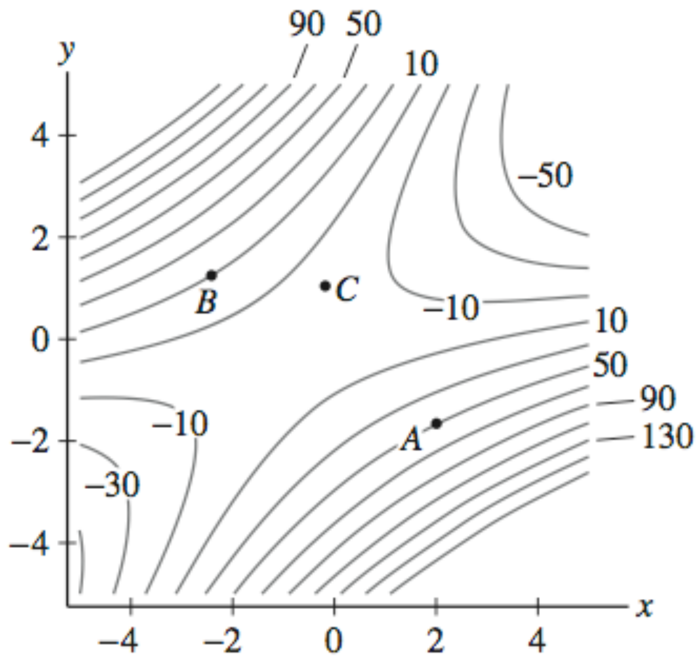


FIGURE 8 Contour map of $f(x, y)$.

a) Estimate f_x and f_y at the point A.

b) Starting at point B, in which direction does f increase most rapidly?

c) At which of A, B, or C is f_y smallest?

2. Refer to the following contour graph of $f(x,y)$.

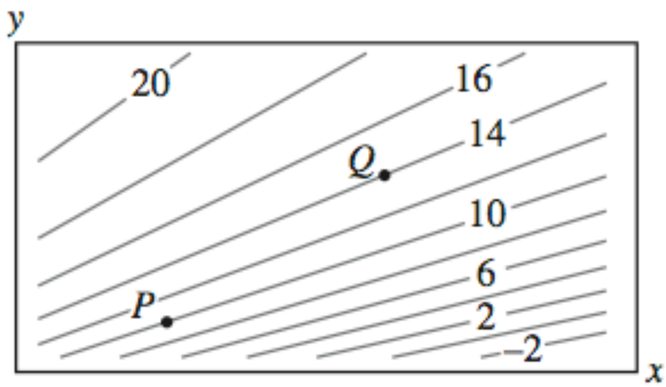


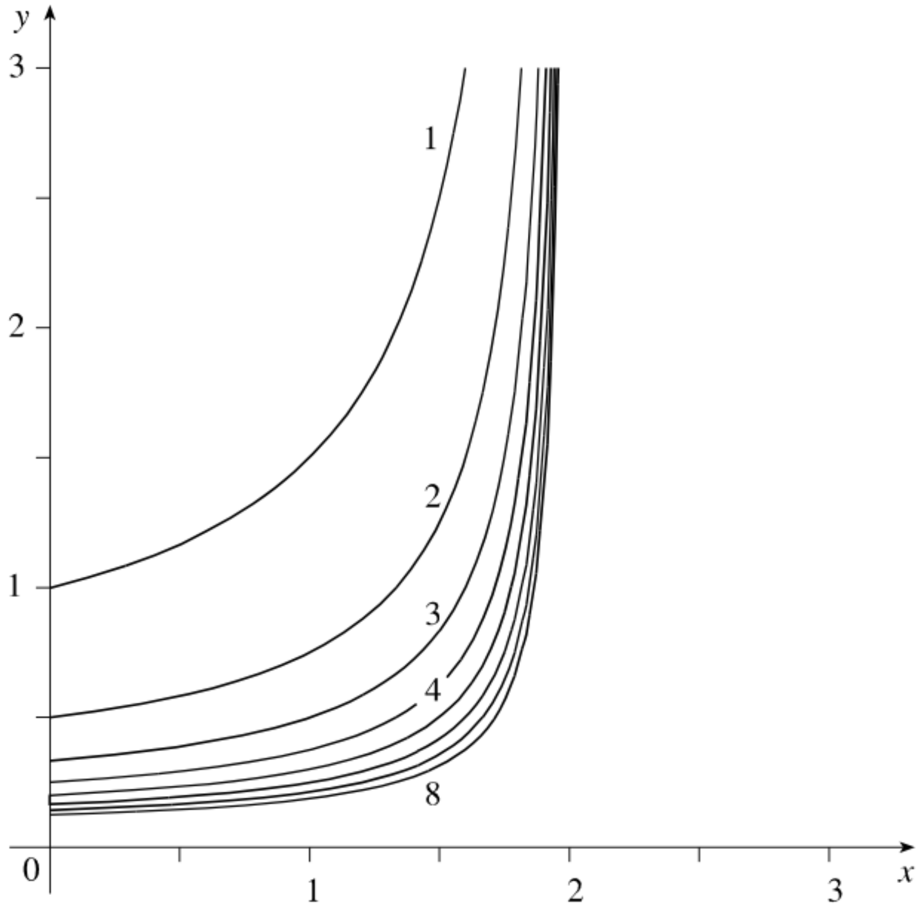
FIGURE 9 Contour interval 2.

a) Explain why f_x and f_y are both larger at P than at Q.

b) Explain why $f_x(x,y)$ is an increasing function of y. That is, for any x, $f_x(x,b_1) > f_x(x,b_2)$ whenever $b_1 > b_2$.

Math 213 - 10.3 - Mixed Partials

The level curves of a function $z = f(x, y)$ are given below.



Use the level curves of the function to decide the signs (positive, negative, or zero) of the derivatives $f_{xx}, f_{yy}, f_{xy}, f_{yx}$, of the function at the point $\left(\frac{3}{2}, \frac{1}{2}\right)$